HOMEOMORPHISM AND DIFFEOMORPHISM GROUPS OF NON-COMPACT MANIFOLDS WITH THE WHITNEY TOPOLOGY

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In this talk we discuss topological properties of homeomorphism and diffeomorphism groups of non-compact manifolds with the Whitney topology [1].

The symbol $(\square^\omega l_2, \boxdot^\omega l_2)$ denotes the pair of countable (box, small box) products of l_2. It is known that the small box product $\boxdot^\omega l_2$ is homeomorphic to $l_2 \times \mathbb{R}^\infty$, the product of l_2 and the direct limit $\mathbb{R}^\infty$ of the tower

$$\mathbb{R}^1 \subset \mathbb{R}^2 \subset \mathbb{R}^3 \subset \cdots .$$

For a non-compact n-manifold M, let $\mathcal{H}(M)$ denote the group of homeomorphisms of M endowed with the Whitney topology. It includes the normal subgroup $\mathcal{H}_c(M)$ consisting of homeomorphisms with compact support. We have obtained the following results:

(1) For any dimension n, the subgroup $\mathcal{H}_c(M)$ is paracompact and locally contractible, and the identity component $\mathcal{H}_0(M)$ of $\mathcal{H}(M)$ is an open normal subgroup in $\mathcal{H}_c(M)$. This induces the topological factorization $\mathcal{H}_c(M) \approx \mathcal{H}_0(M) \times \mathcal{M}_c(M)$ for the mapping class group $\mathcal{M}_c(M) = \mathcal{H}_c(M)/\mathcal{H}_0(M)$ with the discrete topology.

(2) For any non-compact surface M, the pair $(\mathcal{H}(M), \mathcal{H}_c(M))$ is locally homeomorphic to $(\square^\omega l_2, \boxdot^\omega l_2)$ at the identity id_M of M. Thus the subgroup $\mathcal{H}_c(M)$ is an $(l_2 \times \mathbb{R}^\infty)$-manifold.

For a non-compact smooth n-manifold M, let $\mathcal{D}(M)$ denote the group of diffeomorphisms of M endowed with the Whitney C^∞-topology. It includes the normal subgroup $\mathcal{D}_c(M)$ of all diffeomorphisms with compact support.

(3) For any dimension n, the pair $(\mathcal{D}(M), \mathcal{D}_c(M))$ is locally homeomorphic to $(\square^\omega l_2, \boxdot^\omega l_2)$ at id_M. Hence the subgroup $\mathcal{D}_c(M)$ is a topological $(l_2 \times \mathbb{R}^\infty)$-manifold

In [2], [3] we have shown that both the pairs $(\mathcal{H}(\mathbb{R}), \mathcal{H}_c(\mathbb{R}))$ and $(\mathcal{D}(\mathbb{R}), \mathcal{D}_c(\mathbb{R}))$ are homeomorphic to $(\square^\omega l_2, \boxdot^\omega l_2)$.

The Whitney topology on $\mathcal{H}_c(M)$ for $n = 1, 2$ and the Whitney C^∞-topology on $\mathcal{D}_c(M)$ coincide with the group direct limit topology. From this point of view, in [4], [5] we have studied topological types of the identity component $\mathcal{H}(M)_0$ for $n = 2$ and $\mathcal{D}(M)_0$ for any n.

References

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