Homology 3– and 4–manifolds: A survey

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This will be a survey talk on how the topology of *homology 3– and 4–manifolds* differs from that of higher dimensional homology manifolds, mostly from the point of view of the *Bing Topology School*. We shall be mostly considering *generalized manifolds* X, hence finite-dimensional Euclidean neighborhood retracts which satisfy Poincaré Duality. We shall address several essential questions related to this class of spaces:

1. Is every 3–dimensional (resp. 4–dimensional) homology manifold of *Lebesgue covering* dimension 3 (resp. 4)?
2. Is every generalized 3-manifold X^3 an *acyclic*, or even *cell-like* image of a topological 3-manifold M^3, $f : M^3 \to X^3$?
3. Is then $f : M^3 \to X^3$ approximable by homeomorphisms?
4. If not, is then X^3 at least a factor of a topological 4-manifold N^4, in particular is $X \times R \cong R^4$?

Ad 1. This is a very nice result from dimension theory with a beautiful proof due to Walsh. Analogous question about homology 4-manifolds is still open. We shall describe a geometric criterion due to Mitchell, Repovš and Ščepin for finite dimensionality in this latter case.

Ad 2. This is very much connected to the *Poincaré Conjecture*. Modulo that, we shall describe techniques and results concerning several partial resolution theorems (Brin, Bryant, Lacher, McMillan, Repovš, Thickstun).

Ad 3. This is the 3-dimensional analogue of the Edwards *Disjoint Disks Theorem* for topological ($n \geq 5$)-manifolds. We shall describe techniques and results concerning DDP-type theorems (Bing, Daverman, Eaton, Lacher, Lambert, Repovš, Sher, Starbird, Thickstun).

Ad 4. This is related to the classical *Moore Conjecture*. We shall describe techniques and results concerning Moore-type theorems (Daverman, Jakobsche, Repovš).

We shall also present some peculiar examples of homology manifolds over certain groups of coefficients, which *fail* to be homology manifolds over certain other groups of coefficients – this question arises in cohomological dimension theory, which is very closely related to some of the problems described above.